Code No: 151AA

R18

JAWAHARLAL NEHRU TECHNOLOGICAL UNIVERSITY HYDERABAD
B. Tech I Year I Semester Examinations, September - 2023
MATHEMATICS - I
(Common to CE, EEE, ME, ECE, CSE, EIE, IT, MCT, MMT, ECM, AE, MIE, PTM, CSBS, CSIT, ITE, CE(SE), CSE(CS), CSE(AI&ML), CSE(DS), CSE(IOT), CSE(N), TTE, AI&DS, AI&ML, CSD)

Time: 3 Hours **Max. Marks: 75**

Note: i) Question paper consists of Part A, Part B.
ii) Part A is compulsory, which carries 25 marks. In Part A, answer all questions.
iii) In Part B, Answer any one question from each unit. Each question carries 10 marks and may have a, b as sub questions.

PART – A

(25 Marks)

1.a) What is diagonally dominant for Gauss Seidel Iteration Method? Explain with an example. [2]

b) Find the normal form of the following matrix, $A=\begin{bmatrix} 2 & 3 & -1 \\ 1 & -1 & -2 \\ 3 & 1 & 3 \end{bmatrix}$. [3]

c) State Cayley-Hamilton Theorem. [2]

d) Find the nature of the quadratic form, $Q(x,y,z) = x^2 + 5y^2 + z^2 + 2xy + 2yz + 6zx$. [3]

e) Test the convergence of the series $\sum \frac{1}{\left(1+\frac{1}{n}\right)^{n^2}}$. [2]

f) Give three examples for the Convergent, Divergent and Oscillatory series. [3]

g) Verify Rolle's theorem for the following function on the indicated interval. $f(x) = x^2 - 5x + 10$ on $[2, 3]$. [2]

h) Write the geometric means of Lagrange's Mean Value Theorem. [3]

i) Apply Jacobian to verify whether the following functions are functionally dependent, or not.
$u = \frac{x+y}{1-xy}$ and $v = \tan^{-1} x + \tan^{-1} y$ [2]

j) Find the total differential coefficient of the function x^2y with respect to x where $x^2 + xy + y^2 = 1$. [3]

PART - B

(50 Marks)

2. For what values of λ and μ, the system of linear equations:
$$x + y + z = 6$$
$$x + 2y + 5z = 10$$
$$2x + 3y + \lambda z = \mu$$
has (a) a unique solution; (b) no solution; (c) infinite. [10]

OR

3.a) Consider the matrix, $A = \begin{bmatrix} 1 & 1 & -1 & 0 \\ 2^2 & 2^2 & -3 & 1 \\ p & 2 & 2 & 2 \\ 3^2 & 3^2 & p & 3 \end{bmatrix}$. Then, can you determine the values of p such that $\text{rank}(A) = 3$? If not, explain why?

b) Use Gauss Elimination process to solve the system $\begin{bmatrix} 1 & -1 & 1 & 1 \\ 1 & 1 & -1 & 1 \\ 1 & 1 & 1 & -1 \\ 1 & 1 & 1 & 1 \end{bmatrix} \begin{bmatrix} u_1 \\ u_2 \\ u_3 \\ u_4 \end{bmatrix} = \begin{bmatrix} 2 \\ -4 \\ 4 \\ 0 \end{bmatrix}$. [5+5]

4. Given: $A = \begin{bmatrix} 1 & 1 & 1 & 0 \\ 1 & 1 & 0 & 1 \\ 1 & 0 & 1 & 1 \\ 0 & 1 & 1 & 1 \end{bmatrix}$ then,

a) Find the eigenvalues of A.

b) Write the quadrature form corresponding to A. [5+5]

OR

5. Find the characteristic equation of the matrix $A = \begin{bmatrix} 2 & 1 & 1 \\ 0 & 1 & 0 \\ 1 & 1 & 2 \end{bmatrix}$. Verify Cayley-Hamilton theorem and hence prove that:

$A^8 - 5A^7 + 7A^6 - 3A^5 + A^4 - 5A^3 + 8A^2 - 2A + I = \begin{bmatrix} 8 & 5 & 5 \\ 0 & 3 & 0 \\ 5 & 5 & 8 \end{bmatrix}$. [10]

6.a) Discuss the convergence of the series: $\frac{x^2}{2\log 2} + \frac{x^3}{3\log 3} + \frac{x^4}{4\log 4} + \cdots$

b) Test the convergence of $\sum_{n=1}^{\infty} \frac{n!3^n}{n^n}$. [5+5]

OR

7.a) Test the following series for convergence: $\sum \frac{4\cdot 7\cdot \ldots\cdot(3n+1)}{1\cdot 2\cdot \ldots\cdot n} x^n$

b) Test the convergence of $\sum_{n=1}^{\infty} \frac{n^2 + 3n + 5}{2n^2 + 5n + 7}$. [5+5]

8.a) If n is a positive integer, prove that $2^n \Gamma\left(n + \frac{1}{2}\right) = 1 \cdot 3 \cdot 5 \cdots (2n+1)\sqrt{\pi}$.

b) If n is a positive integer and $m > -1$, the show that $\int_0^1 x^m (\log x)^n dx = \frac{(-1)^n\, n!}{(m+1)^{n+1}}$. [5+5]

OR

9.a) Prove that $\beta(m,n) = \frac{\Gamma(m)\Gamma(n)}{\Gamma(m+n)}$.

b) Verify $\int_0^{\pi/2} \frac{\sin^{2m-1}\theta \cos^{2n-1}\theta\, d\theta}{(a\sin^2\theta + b\cos^2\theta)^{m+n}} = \frac{\beta(m,n)}{2a^m b^n}$ is true or not. [5+5]

10. The plane $x + y + z = 1$ cuts the cylinder $x^2 + y^2 = 1$ in an ellipse. Apply Lagrange's multipliers method to find the points on the ellipse that lie closest to and farthest from the origin. [10]

OR

11. Discuss the maxima and minima of $4xy - 2x^2 - 2y^2 + y^4 + x^4$. [10]

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